Application of Network RTK Positions and Geometric Constraints to the Problem of Attitude Determination Using the GPS Carrier Phase Measurements

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Abstract

Nowadays, navigation is an unavoidable fact in military and civil aerial transportations .The Global Positioning System (GPS) is commonly used for computing the orientation or attitude of a moving platform. The relative positions of the GPS antennas are computed using the GPS code and/or phase measurements. To achieve a precise attitude determination, Carrier phase observations of GPS requiring the phase ambiguity resolution has been utilized. The more accurate the coordinates, the more accurate the attitude parameters will be. Attitude parameters are derived from the computed coordinates. Here, attitude parameters are computed by carrier beat phases of four single frequency GPS receivers. The problem of GPS attitude determination is an ill-posed problem if only GPS carrier phases are used. This is because the number of unknown parameters is always larger than the number of observations when the relative positions of the GPS antennas are computed. In this research, carrier beat phases of four single frequency GPS receivers are used to determine the orientation of a platform whose attitude parameters are already known. Observations are made for 10 minutes. In this research, two sets of constraints are used to fix the rank deficiency of the problem. The first consists of the Real Time Kinematic (RTK) coordinates of the GPS antennas. Fixed antennas to the moving body help add five additional constraints (second set) to the problem. These constraints increase the redundancy and make the least-squares estimation of the attitude parameters possible. Since the application of regularization methods contaminates the solution with regularization errors, application of the proposed constraints is superior to regularization techniques. This is practically shown through the comparison of the computed attitude parameters, a similar set of results which is derived using the Moore-Penrose algorithm as a regularization technique, and the reference values of these parameters which are provided through an independent research.

According to the obtained results, 59 seconds is required to fix the ambiguity parameters. In other words, to reduces the accuracy of the float ambiguities to less than 1.0 cycle, their initial estimate should be updated by the next 58 measurement epochs. Then the ambiguity parameters are rounded to their nearest integer number. On average, the least squares estimate of the yaw parameter is $\overline{y} = 51.700^0$ with the standard deviation of $\pm 0.0171^0$. The average estimate of pitch is $\overline{p} = 39.168^0$ with the standard deviation of $\pm 0.0154^0$. Finally, on average, the least squares estimate of the roll is $\overline{r} = 26.153^0$ with the standard deviation of $\pm 0.0137^0$. Computed attitudes have been compared to their known values.

By the new definition of the body frame given in this study, least-squares estimation of the attitude parameters would be possible even if only three GPS antennas are used. Computing the transformation parameters between the new and conventional body frames, attitude angles can be transformed to any conventional frame. The proposed method of this research is superior to the others. The computed biases represent the integrity of determination and corroborate usage of inner constraints and weighted parameters to resolve the rank deficiency of the problem.

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1. Introduction

The term attitude refers to the orientation of a platform with respect to a specific reference frame. Therefore, the attitude of a platform is defined by three rotations about the coordinate axes of a given reference frame. Attitude determination using GPS is based on interferometric technique. As the result, at least two GPS antennas are required for this purpose. In this method, the vector of the relative position of GPS antennas is firstly computed using the carrier phase or code differences. Attitude parameters are then derived from computed relative positions. For precise determination of attitude parameters, application of carrier phase measurements is inevitable. Ambiguity resolution is one of the main challenges in this respect. The method of LAMBDA or one it's modifications is normally used for this purpose [1-5].

Determining the attitude of a moving platform using GPS has been the subject of extensive researches. Application of GPS for attitude determination was firstly proposed by Spinney, 1976 [6]. The first attitude determination results using the GPS carrier phase measurements were reported by Evans who proposed a method to measure attitude angles of a platform with a single antenna that periodically rotated in a plane [7]. The first prototype multi-antenna GPS receiver was manufactured in 1988 and tested in a dynamic marine environment [8, 9]. According to Wilson and Tonnemacher and Comp, the antenna configuration is important when 3 or 4 antennas are used to solve the problem [10] [11]. A multi-antenna GPS system consisting of multiple off-the-shelf GPS sensors has been successfully developed and extensively tested in operational marine environments [12]. Bejeryd compares the application of GPS to a navigation grade Inertial Navigation System (INS) in this problem. Two commercial GPS-based attitude determination systems had been tested there on a mobile platform and compared to the INS results. This experience showed that GPS-based attitude determination works well in open areas, but would requires support from additional sensors in urban and forest environments [13]. Dai et al. developed a toolbox for determining the attitude parameters using the GPS code measurements [14]. Giorgi developed a new GNSS based ambiguity-attitude estimation method in which the carrier phase ambiguities and the platform orientation were properly modeled and integrally solved. The main advantage of that novel ambiguity-attitude estimation method was the very high ambiguity resolution performance, even in weak scenarios, i.e., low number of satellites, higher noise levels, and multipath. Moreover, according to this research; the improvement of the accuracy is proportional to the number of deployed antennas [15]. Rokhlin and Even-tzur examined the influence of adding the $L2$ frequency to the $L1$ on initialization time and the precision of solution [16].

Attitude estimation via GNSS measurements has been demonstrated to be a viable technique with a wide spectrum of challenging applications, ranging from terrestrial to maritime (guidance of land vehicles, precise docking of vessels, precision farming), and from air to space (landing assistance, Unmanned Aerial Vehicles, and guidance and control of space platforms). Examples of such applications can be found in [2, 4, 17-30].

In this research, carrier beat phases of four single frequency GPS receivers are used to determine the orientation of a platform whose attitude parameters are already known. The problem of GPS attitude determination is an ill-posed problem when the GPS carrier phase observations are used. This is because the number of unknown parameters is always larger than the number of observations in the corresponding system of simultaneous equations. Here, two sets of constraints are used to fix the rank deficiency of the problem. The first group consists of the Real Time Kinematic (RTK) coordinates of the GPS antennas. Fixed antennas help add five additional constraints to the problem. These increase the redundancy and make the least-squares estimation of the attitude parameters possible. Therefore, unlike other researches; this research does not implement the Moore-Penrose algorithm for computing coordinates and the float ambiguity parameters. Since the application of the Moore-Penrose algorithm, or any other regularization method, contaminates the solution with regularization errors; the proposed method of this research is superior to the others. This is practically shown through the comparison of the computed attitude parameters and a similar set of results which is derived using the Moore-Penrose algorithm with the known values of these parameters [12]. By the new definition of the body frame which is given in this research, least-squares estimation of the attitude parameters would be possible even if only three GPS antennas are used. Computing the transformation parameters which are required for transforming the attitude angles to a conventional body frame is always possible.

The next section of this paper introduces the theoretical background of the problem. The third section is devoted to the proposed method of this research. The proposed method has been applied to the GPS carrier phase measurements which have been already made on a platform whose attitude parameters are precisely known. Section four discusses on the corresponding numerical results. The concluding remarks are given in the last section of this paper.

2. Fundamentals of the GPS Attitude Determination

To determine the orientation of a moving platform by GPS, the relative positions of at least three GPS antennas are required. Instantaneous attitude of the body is then defined by the rotations of a coordinate frame which is fixed to the body (body frame) about the coordinate axes of another frame which is independent of the body (local level frame). The corresponding rotations are known as roll(r), pitch (p) and yaw (y) which are rotations about the Y, X and Z axes, respectively (see for example Lu, 1995 for further details).

Since the GPS code pseudo-ranges are not precise, carrier beat phases are used to compute the relative positions. The corresponding observation equation is as follows,

$$\varphi_A^i = \rho_A^i + c\left(dt_A - dt^i\right) + T_A^i - I_A^i + \lambda N_A^i + \varepsilon \quad (1)$$

where φ_A^i is the carrier beat phase measurement in meter, ρ_A^i is the geometric distance of satellite i and antenna A, c is the velocity of light, dt^i is the satellite clock error, dt_A is the receiver clock error, T_A^i is the tropospheric error in meters, I_A^i is the ionospheric error in meters, λ is the carrier wave length, N_A^i is the initial phase ambiguity and ε is the observation error.

Since the double difference initial phase ambiguity is an integer quantity [31], the double difference carrier phase measurements are used to compute the antennas' relative positions. In addition, in GPS attitude determination, distances of the GPS antennas are a few meters. Consequently, the corresponding atmospheric errors are almost the same. Therefore, the double difference carrier phase observation equation for satellites i and j and antennas A and B can be written as follows [32],

$$\begin{aligned}\Delta\nabla\varphi_{AB}^{ij} &= \Delta\varphi_{AB}^i - \Delta\varphi_{AB}^j = \left(\varphi_A^i - \varphi_B^i\right) - \left(\varphi_A^j - \varphi_B^j\right) \\ &= \rho_A^i - \rho_B^i - \rho_A^j + \rho_B^j + \lambda\Delta\nabla N_{AB}^{ij} + \Delta\nabla\varepsilon_{AB}^{ij}\end{aligned} \quad (2)$$

Various methods have been developed for resolving the integer values of the ambiguity parameters $\Delta\nabla N$. Many of them employ "On-The-Fly" (OTF) techniques. The Ambiguity Function Method (AFM) [33], Least Squares Ambiguity Search Technique (LSAST) [34], Fast Ambiguity Resolution Approach (FARA) [35], and Least-Square Ambiguity Decorrelation Adjustment (LAMBDA) [36] are some examples.

In matrix notation, the linearized form of the simultaneous system of observation equations is as follows,

$$\mathbf{v} + \mathbf{l} = \mathbf{A}_1\mathbf{x} + \mathbf{B}_1\mathbf{z}, \quad \mathbf{P} = \mathbf{C}_\mathbf{l}^{-1} \quad (3)$$

where, $\mathbf{l}$ is the vector of double difference carrier phases, $\mathbf{v}$ is the corresponding vector of residuals, $\mathbf{x}$ is the vector of coordinates, $\mathbf{z}$ consists of the integer ambiguities, $\mathbf{A}_1$ and $\mathbf{B}_1$ are the coefficient matrices and $\mathbf{C}_\mathbf{l}$ is the fully populated variance-covariance matrix of the double difference carrier phase measurements.

GPS-systems usually use an Earth Centered Earth Fixed (ECEF) coordinate system named WGS-84. When determining attitude a local coordinate system is needed. This normally coincides with the north, east and up or down directions. Examples of such systems are the right handed East, North, Up

(ENU) and North, East, Down (NED) coordinate systems [13].

The local ENU coordinate system is a coordinate frame fixed to the earth's surface. Based on the WGS-84 ellipsoid model, its origin and axes are defined as the following [37]:

1. The origin is arbitrarily fixed to a point on the earth's surface.

2. The X-axis points toward the ellipsoid north (geodetic north).

3. The Y-axis points toward the ellipsoid east (geodetic east).

4. The Z-axis points upward along ellipsoid normal.

ECEF coordinates can be transformed to the East, North, Up (ENU) coordinates by two rotations (Buist, 2013):

$$\begin{bmatrix} E \\ N \\ U \end{bmatrix} = \mathbf{R}_x\left(\frac{\pi}{2}-\varphi\right)\mathbf{R}_z\left(\frac{\pi}{2}+\lambda\right)\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (4a)$$

$$C_{ECEF}^{ENU} = \mathbf{R}_x\left(\frac{\pi}{2}-\varphi\right)\mathbf{R}_z\left(\frac{\pi}{2}+\lambda\right) \quad (4b)$$

$$\mathbf{C}_{ECEF}^{ENU} = \begin{pmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\varphi\cos\lambda & -\sin\varphi\sin\lambda & \cos\varphi \\ \cos\varphi\cos\lambda & \cos\varphi\sin\lambda & \sin\varphi \end{pmatrix} \quad (4c)$$

In Eq. (4), λ and φ are the longitude and latitude of the GPS antenna which is the origin of the local level frame (see Fig. 1).

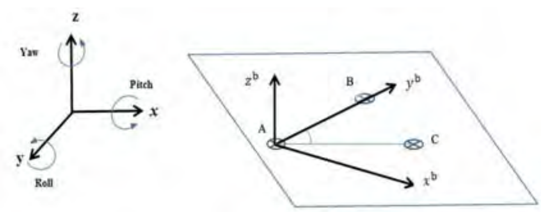


Fig. 1: Local level frame (x, y, z) versus body frame (x^b, y^b, z^b)

The local level coordinates $(x, y, z)^T$ can be transformed to the body frame by the transformation below,

$$\begin{pmatrix} x^b \\ y^b \\ z^b \end{pmatrix} = \mathbf{R}_2(r)\mathbf{R}_1(p)\mathbf{R}_3(y)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{R}(y,p,r)\begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (5)$$

Here, $(x^b, y^b, z^b)^T$ are the body frame coordinates of a GPS antenna. Moreover, using the abbreviations $s(.)$ and $c(.)$ for the sine and cosine functions; transformation matrix $\mathbf{R}(y,p,r)$ is,

$$\mathbf{R}(y,p,r) = \begin{pmatrix} c(r)c(y)-s(r)s(p)s(y) & c(r)s(y)+s(r)s(p)c(y) & -s(r)c(p) \\ -c(p)s(y) & c(p)c(y) & s(p) \\ s(r)c(y)+c(r)s(p)s(y) & s(r)s(y)-c(r)s(p)c(y) & c(r)c(p) \end{pmatrix} \quad (6)$$

Equation (5) is the mathematical model which is used in the least-squares approach to the problem of attitude determination. Using $\mathbf{l}_i^{'} = (x_i^b, y_i^b, z_i^b)^T$ and $\mathbf{l}_i = (x_i, y_i, z_i)^T$, the linearized form of this implicit model is as follows [38],

$$\mathbf{F}_i\hat{\boldsymbol{\delta}} + \begin{pmatrix} \mathbf{E}_i & \mathbf{I}_i \end{pmatrix}\begin{pmatrix} \delta\mathbf{l}_i \\ \delta\mathbf{l}_i^{'} \end{pmatrix} + \mathbf{w}_i = \mathbf{0} \quad (7)$$

Where, $\hat{\boldsymbol{\delta}} = (\delta y, \delta p, \delta r)^T$ is the vector of unknown parameters, $\mathbf{F}_i$ and $\mathbf{E}_i$ are the coefficient matrices and $\mathbf{w}_i$ is the vector of misclosures. The simultaneous system of Eq. (7) is the observation equations for the i^{th} antenna. Least-squares estimate of the unknown parameters is then given by,

$$\hat{\boldsymbol{\delta}} = -\mathbf{N}^{-1}\mathbf{U} = -\left[\sum_{i=1}^{n}\mathbf{F}_i^T\left(\mathbf{E}_i^T\mathbf{C}_{\mathbf{l}_i}\mathbf{E}_i + \mathbf{C}_{\mathbf{l}_i^{'}}\right)^{-1}\mathbf{F}_i\right]^{-1}\left[\sum_{i=1}^{n}\mathbf{F}_i^T\left(\mathbf{E}_i^T\mathbf{C}_{\mathbf{l}_i}\mathbf{E}_i + \mathbf{C}_{\mathbf{l}_i^{'}}\right)^{-1}\mathbf{w}_i\right] \quad (8)$$

In this equation, n is the number of antennas, $\mathbf{C}_{\mathbf{l}_i}$ and $\mathbf{C}_{\mathbf{l}_i^{'}}$ are the variance-covariance matrices of the observations. The characteristic feature of this method is the feasibility of adjusting the observational errors [12].

3. Methodology

GPS attitude determination is an ill-posed problem when GPS carrier phase observations are used. This is because the number of unknown parameters is always larger than the number of observations when the relative positions of the GPS antennas are computed. Regularization techniques are normally used to solve this problem. The Moore-Penrose algorithm which is in fact a Truncated Singular Value Decomposition (TSVD) solution [39] is commonly used for this purpose [40]. Application of the Moore-Penrose algorithm, or any other regularization techniques, results a solution which is contaminated by regularization errors.

To solve this problem, two sets of constraints are suggested here. Application of these constraints not only fixes the rank deficiency of the problem but also makes the least-squares estimation of the attitude parameters possible. The first group consists of the Real Time Kinematic (RTK) coordinates of the GPS antennas. It is practically established that an accuracy of 1 to 3 centimeters is achievable in RTK positioning of a moving receiver when real time corrections are available from a RTK network. More specifically, accuracies of 1 to 2 cm for the horizontal coordinates and 3 cm for the vertical one are normally achieved [41]. RTK coordinates of the GPS antennas can be used as a set of weighted parameters in estimating the solution of the least-squares problem (3). In matrix notation,

$$\mathbf{v}_x + \mathbf{l}_x = \mathbf{x}, \quad \mathbf{P}_x = \mathbf{C}_{\mathbf{l}_x}^{-1} \tag{9}$$

Here, $\mathbf{C}_{\mathbf{l}_x}$ is a diagonal matrix whose elements are the formal accuracies of the RTK coordinates. The accuracies of 2 cm for x & y and 3 cm for z components have been adopted in this research. Application of these constraints to the simultaneous system of observation equations (3) fixes the rank deficiency of the problem. Nevertheless, the redundancy is zero. Therefore, it is not possible to compute the attitude of the body using the least-squares approach which was outlined above.

Additional constraints can be added to the systems of observation equations (3) and (9). Since the GPS antennas are fixed to the body, their relative position remains fixed. Consequently, the relative position of the centroid of the setup, used to compute the attitude parameters, also remains fixed with respect to the GPS antennas. Therefore, the following constraints can be added to the above systems of observation equations,

$$ndx_G = \sum_{i=1}^{n} dx_i = 0 \tag{10a}$$

$$ndy_G = \sum_{i=1}^{n} dy_i = 0 \tag{10b}$$

$$ndz_G = \sum_{i=1}^{n} dz_i = 0 \tag{10c}$$

Where, n is the number of GPS antennas in the setup and (x_G, y_G, z_G) are coordinates of the centroid in the local level frame.

Again, since the moving platform is a rigid body (i.e. the relative distance of the antennas is constant) the distance of centroid to every GPS antenna is constant too. Therefore,

$$n\overline{r}_{Gi} = \sum_{i=1}^{n} r_{Gi} = cte, \quad r_{Gi} = \sqrt{(x_G - x_i)^2 + (y_G - y_i)^2 + (z_G - z_i)^2} \tag{11}$$

It is easily seen that the linearized form of Eq. (11) is as follows,

$$\sum_{i=1}^{n} \left(x_i dx_i + y_i dy_i + z_i dz_i \right) = 0 \tag{12}$$

The azimuth of each direction between the centroid of the setup and the GPS antennas is also a constant parameter at each measurement epoch. Therefore, if θ_{Gi} is the azimuth of the direction between the centroid and the antenna i, it is possible to write,

$$n\overline{\theta}_{Gi} = \sum_{i=1}^{n} \theta_{Gi} = cte, \quad \theta_{Gi} = \tan^{-1}\left(\frac{x_i - x_G}{y_i - y_G} \right) \tag{13}$$

Again, it is easily seen that the linearized form of Eq. (13) is as follows,

$$\sum_{i=1}^{n} \left(y_i dx_i - x_i dy_i \right) = 0 \tag{14}$$

By using equations (3), (9), (10), (12) & (14) and the criterion below,

$$\mathbf{v}^T \mathbf{P} \mathbf{v} + \mathbf{v}_x^T \mathbf{P}_x \mathbf{v}_x = \min \tag{15}$$

The least-squares estimate of the unknown parameters $\left(\hat{\mathbf{x}}^T \quad \hat{\mathbf{z}}^T \right)^T$ can be derived. The method of sequential adjustment has been used for this purpose [42]. Using the measurements of the first epoch, an initial estimate for the unknown parameters is computed. This estimate is updated using the measurements of the second epoch. The new estimate of the unknown parameters is updated using the measurements of the next epoch, etc. As soon as the accuracy of an ambiguity parameter is less than 1.0 cycle, it is fixed to the nearest integer value. This is the rounding method for ambiguity resolution [43]. Ambiguity resolution increases the accuracy of estimated coordinates. The more accurate the coordinates, the more accurate the attitude parameters will be.

4. Numerical Results

Due to the lack of the sufficient situations and facilities for implementing required setup to obtaining kinematic and static observations, we had to use the available static data set. So, in this research, carrier beat phases of four single frequency GPS receivers are used to determine the orientation of a platform whose attitude parameters are already known [14]. Observations are made for 10 minutes and in static mode. The sampling rate of measurements in 1 Hz. Initial coordinates of the GPS antennas and the known attitude parameters for this setup are given in Table 1.

Table 1: Initial coordinate and the known attitudes of the setup whose observations are used in this research, [14]

Ant No.	$X(m)$	$Y(m)$	$Z(m)$	Attitude parameters
A	3991096.821	563014.827	4927065.332	r = 51.6656^0 p = 26.1822^0 y=-39.1834^0
B	3991081.107	562998.402	4927061.001	
C	3991081.400	563019.756	4927065.029	
D	3991093.445	563007.247	4927059.915	

Using known distances of the GPS antennas, the antennas' coordinates can be computed in a body frame. To analyze the efficiency of the method of this research, at first the attitude parameters are computed using the same body frame as Dai et al. (2010). For this purpose,

Antenna A has been selected as the origin of this frame. The axis y^b of this frame passes through Antenna B. The axis z^b is perpendicular to the surface of the setup and the coordinate system is a right handed frame. The computed body frame coordinates of the antennas is given in Table 2.

Table 2: Body frame coordinates of the GPS antennas in the setup of this research

Ant No.	$x^b(m)$	$y^b(m)$	$z^b(m)$
A	0	0	0
B	0	23.2148	0
C	14.6511	6.999	0
D	-3.0597	8.7469	-3.6491

Because of the small distances between the GPS antennas, every two antennas can be used for setting up a baseline. Therefore, six baselines have been set up. Observational errors of the corresponding double difference carrier phases are simultaneously adjusted using the method of section 3. According to the obtained results, 59 seconds is required to fix the ambiguity parameters. In other words, to reduces the accuracy of the float ambiguities to less than 1.0 cycle, their initial estimate should be updated by the next 58 measurement epochs. Then the ambiguity parameters are rounded to their nearest integer number. Table 3 and Table 4 compare the resolved ambiguities to their known values. Table 3 gives the resolved ambiguities and Table 4 provides the corresponding known values. According to the obtained results, 82 % of the ambiguity parameters are perfectly resolved. Moreover, the existing bias on the others does not exceed 1.0 cycle.

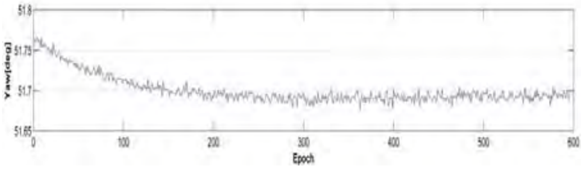
Table 3: Resolved values for ambiguities

Baseline	Resolved ambiguities (in Cycles)							
	N^{16}	N^{110}	N^{116}	N^{117}	N^{121}	N^{122}	N^{126}	N^{130}
AB	0	-1	-1	-1	-14376916	0	1	-3
AC	-8	-8	6	-5	-3	-8	2	-10
AD	-4	-4	1	-7	2	0	-1	-9

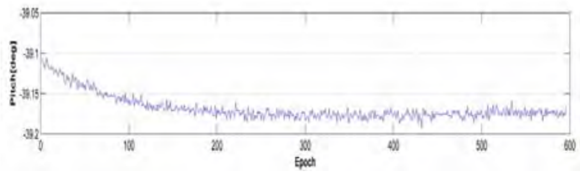
Table 4: Known values of the ambiguities

Baseline	Known ambiguities (in Cycles)							
	N^{16}	N^{110}	N^{116}	N^{117}	N^{121}	N^{122}	N^{126}	N^{130}
AB	0	-1	-1	-1	-14376916	0	1	-3
AC	-8	-8	5	-5	-3	-8	1	-10
AD	-4	-5	0	-7	1	0	-1	-9

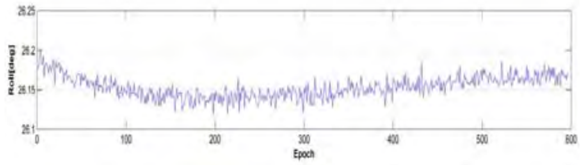
The attitude parameters are estimated sequentially after the ambiguity parameters have been resolved. Figures 2, illustrates the obtained results.



(a)



(b)



(c)

Fig. 2: Sequentially updated attitude parameters: (a) yaw, (b) pitch and (c) roll.

On average, the least squares estimate of the yaw parameter is $\overline{y} = 51.700^0$ with the standard deviation of $\pm 0.0171^0$. The average estimate of pitch is $\overline{p} = 39.168^0$ with the standard deviation of $\pm 0.0154^0$. Finally, on average, the least squares estimate of the roll is $\overline{r} = 26.153^0$ with the standard deviation of $\pm 0.0137^0$. Computed attitudes have been compared to their known values. Table 5 reports on the corresponding results. In this table, attitude parameters which are computed using the Moore-Penrose algorithm have been compared to their known values too. Parameters $B_{\Delta r^0}$, $B_{\Delta p^0}$ and $B_{\Delta y^0}$ are the existing bias in estimated rotations of the setup.

Table 5: Comparison of computed attitudes to their known values

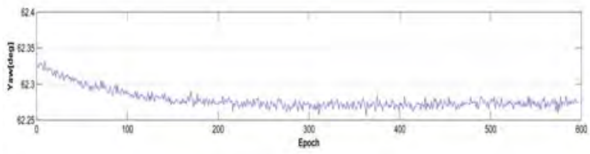
Method \ Bias	$B_{\Delta r^0}$	$B_{\Delta p^0}$	$B_{\Delta y^0}$
This research	-0.0292	0.0154	0.0344
Moore-Penrose algorithm	3.0873	5.9413	-0.4302

By appropriate definition of the body frame, least-squares estimation of the attitude parameters will be possible even if three GPS antennas are used in the setup. For this purpose, the origin of the body frame is transformed to the centroid of the setup. The y^b of this frame is assumed to pass through Antenna 2. The axis z^b is perpendicular to the surface of the setup and the coordinate system is a right handed frame. Coordinates of the GPS antennas are computed in the new body frame (see Table 6 for further details).

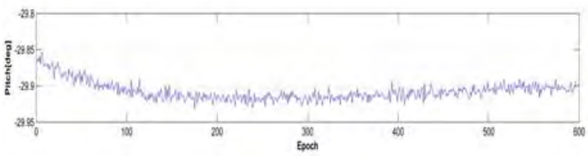
Table 6: Body frame coordinates of the GPS antennas

Antenna No.	$x^b\,(m)$	$y^b\,(m)$	$z^b\,(m)$
centroid	0	0	0
B	0	13.8071	0
C	10.9780	-5.0841	0
A	-4.6751	-8.8255	2.0428
D	-6.3028	0.1025	-2.0428

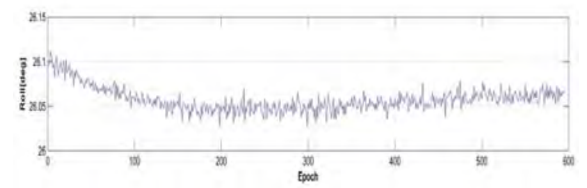
The attitude parameters are computed again using the method of this research. Figure 3 reports on the obtained results. Computing the transformation parameters between the new and conventional body frames, attitude angles can be transformed to the conventional frame.



(a)



(b)



(c)

Fig. 3: Sequentially updated attitude parameters in the new body frame: (a) yaw, (b) pitch and (c) roll.

On average, the least squares estimate of the yaw, pitch and roll are $\overline{y} = 62.278^0$, $\overline{p} = 29.907^0$ and $\overline{r} = 26.057^0$ with the standard deviations of ±0.0134^0, ±0.0124^0 and ±0.0132^0, respectively.

5. Conclusions

Nowadays, Global Navigation Satellite Systems are used to determine the orientation of a moving platform. To increase the accuracy of attitude parameters, application of carrier beat phases is inevitable. The number of unknown parameters is always larger than the number of measurements if carrier beat phases are used. To compute a unique solution for the problem, the pseudo-inverse of the coefficient matrix is normally used.

Unlike other researches, this research does not implement the Moore-Penrose algorithm for computing coordinates and the float ambiguity parameters. Since the application of the Moore-Penrose algorithm, or any other regularization method, contaminates the solution with regularization errors; the proposed method of this research is superior to the others. This is practically shown through the comparison of the computed attitude parameters and a similar set of results which is derived using the Moore-Penrose algorithm with the known values of these parameters. The computed biases represent the integrity of determination and corroborate usage of inner constraints and weighted parameters to resolve the rank deficiency of the problem.

By the new definition of the body frame which is given in this research, least-squares estimation of the attitude parameters would be possible even if only three GPS antennas are used. Computing the transformation parameters which are required for transforming the attitude angles to a conventional body frame is always possible.

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